

Name:

Solutions

Directions: Answer all questions without a calculator:

1. $\lim_{x \rightarrow \pi} (\cos x) = -1$

2. $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

3. $\lim_{t \rightarrow 2^+} \frac{1}{t-2} = \infty$

4. $\lim_{k \rightarrow -1} \frac{|k+1|}{k+1} = \text{undefined}$

5. $\lim_{t \rightarrow \infty} \frac{4t^3 - 13t}{3t^2 - 2t^3} = -2$

6. $\lim_{t \rightarrow \infty} \frac{4t^2 + 9t^3}{2t^3 - 3t^4} = 0$

7. $\lim_{t \rightarrow \infty} \frac{6t^2 + 2t^3}{6t^2 - 7} = \infty$

8. Given $f(t) = \begin{cases} t^2 + 2t - 17 & \text{if } t \leq 2 \\ -2t - 4 & \text{if } t > 2 \end{cases}$, $\lim_{t \rightarrow 2^+} f(t) = -8$

9. Given $g(x) = \begin{cases} \sqrt{3x+6} & \text{if } x \geq 1 \\ -\frac{1}{2}x + \frac{7}{2} & \text{if } x < 1 \end{cases}$, $\lim_{x \rightarrow 1} g(x) = 3$

10. Evaluate $\lim_{x \rightarrow \infty} \frac{\cos x + 7x}{x} = 7$

11. Name the types of discontinuities of the graph of $f(x) = \frac{x^2 - 3x - 18}{x^2 - 8x + 12}$ and tell for what values of x these discontinuities exist.

$$f(x) = \frac{(x-6)(x+3)}{(x-6)(x-2)}$$

$f(x)$ has a removable discontinuity at $x=6$
 $f(x)$ has an asymptotic discontinuity at $x=2$

12. Find a simple left end behavior model for the function $f(x) = e^{-x} + \sin x$

$$g(x) = e^{-x}$$

13. Find a simple right end behavior model for the function $f(x) = e^{-x} + \sin x$

$$g(x) = \sin x$$

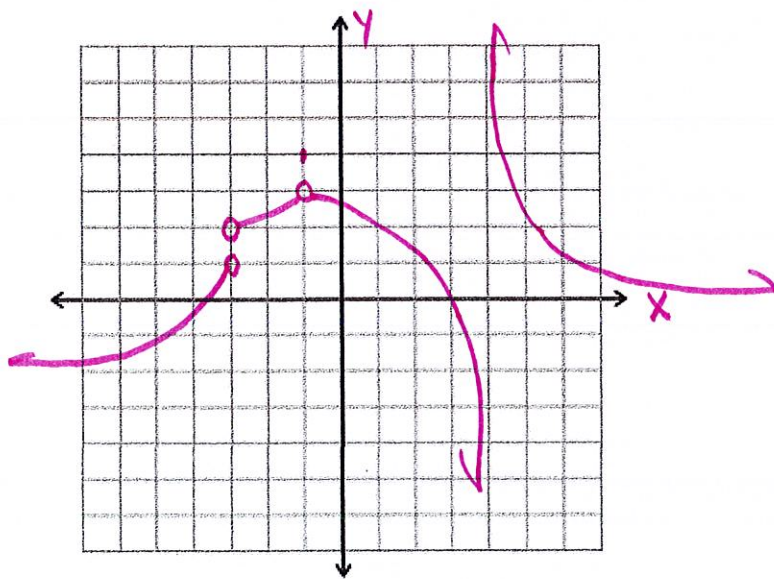
14. Write a power function end behavior model for the function $f(x) = \frac{5x^3 - 9}{2x^2 - x + 3}$

$$g(x) = \frac{5}{2}x$$

15. Sketch a graph of a function $y = f(x)$ that satisfies the stated conditions. Include any asymptotes.

Conditions: $f(-1) = 4$, $\lim_{x \rightarrow -1} f(x) = 3$, $\lim_{x \rightarrow 4^-} f(x) = -\infty$, $\lim_{x \rightarrow 4^+} f(x) = \infty$, $\lim_{x \rightarrow \infty} f(x) = 0$,

$\lim_{x \rightarrow -3^+} f(x) = 2$, $\lim_{x \rightarrow -3^-} f(x) = 1$, & $\lim_{x \rightarrow -\infty} f(x) = -2$



16. Write an extended function for the function $p(x) = \frac{x-1}{x^3-1}$ that is continuous at the point

$$x = 1$$

$$p(x) = \frac{x-1}{(x-1)(x^2+x+1)} \approx \frac{1}{x^2+x+1}$$

$$g(x) = \frac{1}{x^2+x+1}$$

$$\text{or } g(x) = \begin{cases} \frac{x-1}{x^3-1} & \text{if } x \neq 1 \\ \frac{1}{3} & \text{if } x = 1 \end{cases}$$

$$\frac{(x-5)(x+5)}{x-5} \stackrel{\text{almost}}{=} x+5$$

17. Is the function $f(x) = \begin{cases} \frac{x^2-25}{x-5} & \text{if } x \neq 5 \\ 10 & \text{if } x = 5 \end{cases}$ continuous on its domain?

Yes, because $f(5) = 10$ and $\lim_{x \rightarrow 5} f(x) = f(5) = 10$

18. Find a value for "a" for which the function $f(x) = \begin{cases} 3x+2, & x \geq 3 \\ ax-4, & x < 3 \end{cases}$ is continuous

$$3(3) + 2 = a(3) - 4$$

$$11 = 3a - 4$$

$$15 = 3a$$

$$5 = a$$

19. Which of the following points of discontinuity of $f(x) = \frac{x^2(x-2)(x+1)}{x(x-2)^2(x+1)}$ is not removable?

a. $x=0$

b. $x=2$

c. $x=-1$

d. none of these

20. Find the average rate of change of $p(x) = 2x^2 - x$ over the interval $[2, 4]$

$$\frac{2(4)^2 - 4 - (2(2)^2 - 2)}{4 - 2} = \frac{32 - 4 - (8 - 2)}{2} = \frac{28 - 6}{2} = \frac{22}{2} = 11$$

21. Find the instantaneous rate of change (or slope) of the function $f(x) = 3x^2 - 5x$ at $x = 3$.

$$\begin{aligned} & \lim_{h \rightarrow 0} \frac{3(a+h)^2 - 5(a+h) - (3a^2 - 5a)}{a+h-a} \\ &= \lim_{h \rightarrow 0} \frac{\cancel{3a^2} + 6ah + 3h^2 - \cancel{5a} - 5h - \cancel{3a^2} + \cancel{5a}}{h} \\ &= \lim_{h \rightarrow 0} \frac{6ah + 3h^2 - 5h}{h} \\ &= \lim_{h \rightarrow 0} (6a + 3h - 5) = 6a - 5 \end{aligned}$$

$$\text{At } x=3 \text{ or } a=3$$

$$6a - 5 = 6(3) - 5 = 13$$